Motion Compensation aboard the Seward Johnson for Radars and Lidar

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1 Motion Compensation aboard the Seward Johnson

A **superscript** L will denote a vector's components in the Lidar's motion detection coordinate system. A **superscript** S will denote a vector's components in the Ship's motion detection coordinate system. A **superscript** E will denote a vector's components in the Earth's coordinate system, that is, north, east, down in that order. The phrase "in the coordinate system" means that the vector's components are obtained by projection of the vector along the coordinate axes, that is, by inner product of the vector with the unit vectors that are aligned with the positive direction of each axis. The Lidar's and Ship's coordinate systems have their origins spatially displaced from one another by a fixed separation vector, and they are in fixed orientation relative to one another. The Ship's coordinate system is forward (x), starboard (y) and down (z). Both coordinate systems are assumed to be right handed in the order (x, y, z) which are the names of the axes; (x, y, z) corresponds to and can be replaced by numerical indices $(1, 2, 3)$. Rotation about the x axis in the right-handed sense is called 'roll' ϕ; rotation about the y axis in the right-handed sense is called 'pitch' θ; rotation about the z axis in the right-handed sense is called 'heading' or 'yaw' ψ.

Both Lidar and Ship coordinate systems are translating relative to the Earth's coordinate system which is fixed relative to the Earth's lithosphere. A given velocity v is denoted by v^S when its components are in the Ship's coordinate system and by v^L when its components are in the Lidar's coordinate system and by v^E when its components are in the Earth's coordinate system. Both the Lidar's and Ship's coordinate systems are rotating relative to a coordinate system that is fixed relative to the Earth's lithosphere by angular rate denoted by Ω^S when its components are in the Ship's coordinate system and by Ω^L when its components are in the Lidar's coordinate system. Ω^L and Ω^S are the same vector because the ship is a rigid body.

A **subscript** L denotes a quantity measured by the Lidar's motion detection system, and a **subscript** S denotes a quantity measured by the Lidar's motion detection system. No subscript appears on quantities calculated from the measured quantities. Although Ω^L and Ω^S are the same vector, because of measurement errors Ω^S_S and Ω^L_L differ; that difference is a function of time because of random measurement errors. The analogous statement is **not** true of v^E_S and v^E_L because the velocity at the Lidar differs from the velocity at the origin of the coordinate system of the Ship's motion detection system.

1.1 Measured Quantities and Units

If the data is in units other than are stated here, then the data should be changed to the units here. In particular, angles and angular rates are probably in degrees and degrees per second, respectively.

$\mathbf{\Omega}^L_L$ is the vector of angular rates in radians per second measured by the Lidar's motion detection system.

$\mathbf{\Omega}^S_S$ is the vector of angular rates in radians per second measured by the Ship's motion detection system.

v^E_L is the velocity vector in meters per second measured by the Lidar's motion detection system with its components in the Earth's coordinate system (north,east,down).

v^E_S is the velocity vector in meters per second measured by the Ship's motion detection system with its components in the Earth's coordinate system (north,east,down).

$(\phi_L, \theta_L, \psi_L)$ The Euler angles from the Lidar's motion detection system determined from integration of the angular rates.

$(\phi_S, \theta_S, \psi_S)$ The Euler angles from the Ship's motion detection system determined from integration of the angular rates

t_L is the sequence of times in seconds at which $\mathbf{\Omega}_L^L$ and v_L^E are measured.

t_S is the sequence of times in seconds at which $\mathbf{\Omega}_S^S$ and v_S^E are measured.

The ideal time to take this data is when the ship moves in two circles to compensate the flux system's magnetic compass for ship magnetism.

1.2 Quantities to be Determined by Regression: $r^S(SL)$ and R

$r^S(SL)$ is the position vector in meters that points from the location on the Ship where the velocity is v_S^E to the location at the Lidar where the velocity is v_L^E. Since it has superscript S, the components of $r^S(SL)$ are in the Ship's motion detection coordinate system.

R is the 3 by 3 coordinate transformation matrix that transforms a vector from the Lidar's coordinate system to the Ship's coordinate system. R is expressed in terms of the Euler angles of that coordinate transformation by

$$R = \begin{bmatrix} \cos\theta\cos\psi & \cos\theta\sin\psi & -\sin\theta \\ -\cos\phi\sin\psi + \sin\theta\cos\psi\sin\phi & \cos\phi\cos\psi + \sin\theta\sin\phi\sin\psi & \cos\theta\sin\phi \\ \sin\phi\sin\psi + \cos\phi\sin\theta\cos\psi & -\cos\psi\sin\phi + \cos\phi\sin\theta\sin\psi & \cos\theta\cos\phi \end{bmatrix} \tag{1}$$

The quantities to be determined by regression are the 3 components of $r^S(SL)$ and the 3 Euler angles. Of course, R is also the matrix of 9 direction cosines formed by the inner products of the unit vectors aligned along the positive axes of the Ship's coordinate system with the unit vectors aligned along the positive axes of the Lidar's coordinate system.

1.3 Equations Used for the Regression.

1.3.1 The rotation matrix R

The time sequences must coincide, but the sample times are not necessarily the same, that is, $t_L \neq t_S$. Assume that the data rates are unequal. If the Lidar has the faster time series, then linearly interpolate the values of $\mathbf{\Omega}_L^L$ and v_L^E to the slower time sequence t_S. If the Ship has the faster time series, then linearly interpolate the values of $\mathbf{\Omega}_S^S$ and v_S^E to the times t_L. If the data are synchronized, then no interpolation is needed.

At every position on the ship, the angular rates are the same because the ship is a rigid body. Therefore, by definition of R, and neglecting the measurement errors,

$$\mathbf{\Omega}_S^S = R\mathbf{\Omega}_L^L \tag{2}$$

This constitutes 3 nonlinear transcendental equations for the 3 unknown Euler angles. It can be solved at each time using a Newton-Rapheson subroutine. The solution will vary with time because of random errors in $\mathbf{\Omega}_S^S$ and $\mathbf{\Omega}_L^L$. It is easier to use all of the time series in a regression subroutine to obtain the Euler angles and their random errors. It may be yet easier to ignore the Euler angle formulation and obtain all 9 components of R by regression.

At this point, we know R. A reality check can be performed. Further, R should be an orthogonal transformation such that its transpose is its inverse. Therefore, test that

$$R^T = R^{-1} \tag{3}$$

Let I be the identity matrix. The test of (3) is that the 9 elements of the matrix

$$RR^T - I \tag{4}$$

should be small compared to unity.

Another way to get R is to use the Euler angles $(\phi_L, \theta_L, \psi_L)$ and $(\phi_S, \theta_S, \psi_S)$ which are functions of time. By definition, any vector U has its Earth, Lidar, and Ship's components related by

$$\begin{aligned} R(\phi_L, \theta_L, \psi_L)\, U^E &= U^L \\ R(\phi_S, \theta_S, \psi_S)\, U^E &= U^S \end{aligned}$$

where $R(\phi_L, \theta_L, \psi_L)$ is (1) with (ϕ, θ, ψ) replaced by $(\phi_L, \theta_L, \psi_L)$ and $R(\phi_S, \theta_S, \psi_S)$ is (1) with (ϕ, θ, ψ) replaced by $(\phi_S, \theta_S, \psi_S)$. Thus,

$$\begin{aligned} U^E &= R^T(\phi_L, \theta_L, \psi_L)\, U^L \\ U^E &= R^T(\phi_S, \theta_S, \psi_S)\, U^S \end{aligned}$$

Eliminating U^E gives $R^T(\phi_L, \theta_L, \psi_L)\, U^L = R^T(\phi_S, \theta_S, \psi_S)\, U^S$, thus,

$$U^S = R(\phi_S, \theta_S, \psi_S)\, R^T(\phi_L, \theta_L, \psi_L)\, U^L$$

from which the definition of R in (1) gives

$$R = R(\phi_S, \theta_S, \psi_S)\, R^T(\phi_L, \theta_L, \psi_L)$$

R should be independent of time whereas $R(\phi_L, \theta_L, \psi_L)$ and $R(\phi_S, \theta_S, \psi_S)$ vary with time. R will vary with time because of random errors in $(\phi_L, \theta_L, \psi_L)$ and $(\phi_S, \theta_S, \psi_S)$.

1.3.2 The velocity at a given point on the ship

Consider a position vector r^S that points from the Ship's coordinate origin where the velocity is v_S^E, as measured by the Ship's motion detection system, to any other location. Let $V^E(r)$ be the velocity of that location on the ship. Since V^E has superscript E, V^E is expressed in the Earth's coordinate system. The velocity $V^E(r^S)$ as determined by the Ship's motion detection system is

$$V^E\left(r^S\right) = v_S^E + \left(\mathbf{\Omega}_S^S \times r^S\right)^E \tag{5}$$

How to calculate $\left(\mathbf{\Omega}^S \times r^S\right)^E$ is given on separate sheets. In particular, the velocity at the Lidar's motion detection system, as determined by the Ship's motion detection system, is

$$V^E\left(r^S\left(SL\right)\right) = v_S^E + \left(\mathbf{\Omega}_S^S \times r^S\left(SL\right)\right)^E \tag{6}$$

where the argument (SL) of $r^S\left(SL\right)$ denotes Ship origin to Lidar origin. Recall that the velocity of that point as measured by the Lidar's motion detection system is v_L^E. Equating the Ship's and Lidar's measured velocities, i.e., $V^E\left(r^S\left(SL\right)\right) = v_L^E$, gives an equation for $r^S\left(SL\right)$, namely,

$$v_L^E = v_S^E + \left(\mathbf{\Omega}_S^S \times r^S\left(SL\right)\right)^E \tag{7}$$

Note that we have neglected the measurement errors in v_L^E and v_S^E; those errors produce error in $r^S\left(SL\right)$, as do errors in R and $\mathbf{\Omega}_S$. Equation (7) constitutes 3 linear algebraic equation for the 3 unknown components of $r^S\left(SL\right)$. It can be solved at each time using either a matrix inversion or an analytically inverted 3 by 3 matrix. The solution will vary with time because of random errors in v_S^E and v_L^E. It is easier to use all of the time series in a regression subroutine to obtain the 3 components of $r^S\left(SL\right)$ and their random errors.

At this point we know $r^S\left(SL\right)$. A reality check can be performed.

1.4 Uses for the Above Results

1.4.1 Correct the Lidar's data using the Ship's motion detection system:

Multiply (2) by R^{-1}. to obtain

$$\mathbf{\Omega}^L = R^{-1}\mathbf{\Omega}_S^S \tag{8}$$

Since we now know $r^S\left(SL\right)$ we use it in (7) to give $v^E\left(r^S\left(SL\right)\right)$ as a substitute for v_L^E from

$$v^E\left(r^S\left(SL\right)\right) = v_S^E + \left(\mathbf{\Omega}_S^S \times r^S\left(SL\right)\right)^E \tag{9}$$

We thereby obtain the quantities needed by the Lidar from the Ship's motion detection, namely v^E at the Lidar from (9) and $\mathbf{\Omega}^L$ from (8).

1.4.2 Correct NOAA/K Doppler velocities using the Ship's motion detection system:

Let $r^S\left(SK\right)$ denote the position vector in meters that points from the location on the ship where the velocity is measured to be v_S^E to the location of the NOAA/K antenna. $r^S\left(SK\right)$ must be measured. Since $r^S\left(SK\right)$ has superscript S, the components of $r^S\left(SK\right)$ are in the Ship's motion detection coordinate system. From (5) the velocity vector at the NOAA/K antenna is

$$V^E\left(r^S\left(SK\right)\right) = v_S^E + \left(\mathbf{\Omega}_S^S \times r^S\left(SK\right)\right)^E \tag{10}$$

$\mathbf{\Omega}_S^S$ is recorded to determine the pointing direction of the NOAA/K antenna. Of course, $\mathbf{\Omega}_L^L$ is also recorded such that $\mathbf{\Omega}_L^L$ can be used to determine the pointing direction of the NOAA/K antenna. It is also possible in the future to use $\mathbf{\Omega}_S$ to correct the pointing of the NOAA/K antenna.

1.4.3 Correct NOAA K Doppler Velocities using the Lidar's motion detection system:

As above, $r^S(SK)$ denotes the position vector in meters that points from the location on the ship where the velocity is v_S^E to the location of the NOAA/K antenna. Solve (7)$v_L^E = v_S^E + \left(\mathbf{\Omega}_S^S \times r^S(SL)\right)^E$ for v_S^E and substitute it into (10)$V^E\left(r^S(SK)\right) = v_S^E + \left(\mathbf{\Omega}_S^S \times r^S(SK)\right)^E$. From (5)$V^E\left(r^S\right) = v_S^E + \left(\mathbf{\Omega}_S^S \times r^S\right)^E$ The velocity vector at the NOAA/K antenna is

$$\begin{aligned} V^E\left(r^S(SK)\right) &= v_L^E - \left(\mathbf{\Omega}_S^S \times r^S(SL)\right)^E + \left(\mathbf{\Omega}_S^S \times r^S(SK)\right)^E \\ V^E\left(r^S(SK)\right) &= v_L^E + \left(\mathbf{\Omega}_S^S \times \left[r^S(SK) - r^S(SL)\right]\right)^E \qquad (11) \end{aligned}$$

Finally, replace the Ship's measurement $\mathbf{\Omega}_S^S$ with the angular rate derived from the Lidar's measurement according to (2), i.e., $\mathbf{\Omega}^S = R\mathbf{\Omega}_L^L$, to obtain

$$V^E\left(r^S(SK)\right) = v_L^E + \left(\left[R\mathbf{\Omega}_L^L\right] \times \left[r^S(SK) - r^S(SL)\right]\right)^E \qquad (12)$$

Note that $r^S(SK) - r^S(SL)$ is the position vector in meters that points from the location where the Lidar measures the velocity v_L^E to the location of the NOAA/K antenna. The components of $r^S(SK) - r^S(SL)$ are in the Ship's coordinate system. Note that this contains the Lidar's measured velocity and angular rate rotated to the Ship's motion detection coordinate system and contains the cross product with $r^S(SK) - r^S(SL)$ performed in the Ship's coordinate system. Although $\mathbf{\Omega}_L^L$ is recorded to determine the pointing direction of the NOAA/K antenna, $\mathbf{\Omega}^S = R\mathbf{\Omega}_L^L$ is calculated to correct the pointing. The reason is that NOAA/K antenna pointing is measured in the ship's coordinate system.

1.4.4 Correct the U. Miami's X- and W-Band Doppler velocities using either the Lidar's or Ship's motion detection system:

The method is the same as above for NOAA/K. Let $r^S(SX)$ and $r^S(SW)$ denote the position vector in meters that points from the location on the ship where the velocity is measured to be v_S^E to the location of the X-band and W-band antennas, respectively. $r^S(SX)$ and $r^S(SW)$ must be measured. $r^S(SX)$ and $r^S(SW)$ are in the Ship's motion detection coordinate system. Replace $r^S(SK)$ in (10) or (12) with either $r^S(SX)$ or $r^S(SW)$ to obtain the respective correction.