0.1 Using Alan Brewer's notes, determine the correct Euler transformation matrix

(The symbols ϕ and θ are interchanged relative to the other file)

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & \sin\theta \\ 0 & -\sin\theta & \cos\theta \end{bmatrix} \text{ last, rotate about the roll axis by } \theta$$

$$B = \begin{bmatrix} \cos\phi & 0 & -\sin\phi \\ 0 & 1 & 0 \\ \sin\phi & 0 & \cos\phi \end{bmatrix} \text{ second rotate about the pitch axis by } \phi$$

$$C = \begin{bmatrix} \cos\psi & \sin\psi & 0 \\ -\sin\psi & \cos\psi & 0 \\ 0 & 0 & 1 \end{bmatrix} \text{ first, rotate about the heading axis by } \psi$$

$$BC = \begin{bmatrix} \cos\phi\cos\psi & \cos\phi\sin\psi & -\sin\phi \\ -\sin\psi & \cos\psi & 0 \\ \cos\psi\sin\phi & \sin\phi\sin\psi & \cos\phi \end{bmatrix}$$

$$D = \begin{bmatrix} \cos\phi\cos\psi & \cos\phi\sin\psi & -\sin\phi \\ -\sin\psi & \cos\psi & 0 \\ \cos\psi\sin\phi & \sin\phi\sin\psi & \cos\phi \end{bmatrix}$$

$$AD = \begin{bmatrix} \cos\phi\cos\psi & \cos\phi\sin\psi & -\sin\phi \\ -\cos\theta\sin\psi + \sin\theta\cos\psi\sin\phi & \cos\theta\cos\psi + \sin\theta\sin\phi\sin\psi & \cos\phi\sin\theta \\ \sin\theta\sin\psi + \cos\theta\cos\psi\sin\phi & -\sin\theta\cos\psi + \cos\theta\sin\phi\sin\psi & \cos\theta\cos\phi \end{bmatrix}$$

$$ABC = \begin{bmatrix} \cos\phi\cos\psi & \cos\phi\sin\psi & -\sin\phi \\ -\cos\theta\sin\psi + \sin\theta\cos\psi\sin\phi & \cos\theta\cos\psi + \sin\theta\sin\phi\sin\psi & \cos\phi\sin\theta \\ \sin\theta\sin\psi + \cos\theta\cos\psi\sin\phi & -\sin\theta\cos\psi + \cos\theta\sin\phi\sin\psi & \cos\theta\cos\phi \end{bmatrix}$$

$$\begin{bmatrix} \cos\phi\cos\psi & \cos\phi\sin\psi & -\sin\phi \\ -\cos\theta\sin\psi + \sin\theta\cos\psi\sin\phi & \cos\theta\cos\psi + \sin\theta\sin\phi\sin\psi & \cos\phi\sin\theta \\ \sin\theta\sin\psi + \cos\theta\cos\psi\sin\phi & -\sin\theta\cos\psi + \cos\theta\sin\phi\sin\psi & \cos\theta\cos\phi \end{bmatrix},$$

$$\text{transpose: } \begin{bmatrix} \cos\phi\cos\psi & -\cos\theta\sin\psi + \sin\theta\cos\psi\sin\phi & \sin\theta\sin\psi + \cos\theta\cos\psi\sin\phi \\ \cos\phi\sin\psi & \cos\theta\cos\psi + \sin\theta\sin\phi\sin\psi & -\sin\theta\cos\psi + \cos\theta\sin\phi\sin\psi \\ -\sin\phi & \cos\phi\sin\theta & \cos\theta\cos\phi \end{bmatrix}$$

Below, matrix times its transpose is verified to be the identity matrix

$$\begin{bmatrix} \cos\phi\cos\psi & \cos\phi\sin\psi & -\sin\phi \\ -\cos\theta\sin\psi + \sin\theta\cos\psi\sin\phi & \cos\theta\cos\psi + \sin\theta\sin\phi\sin\psi & \cos\phi\sin\theta \\ \sin\theta\sin\psi + \cos\theta\cos\psi\sin\phi & -\sin\theta\cos\psi + \cos\theta\sin\phi\sin\psi & \cos\theta\cos\phi \end{bmatrix} \begin{bmatrix} \cos\phi\cos\psi & -\cos\theta\sin\psi \\ \cos\phi\sin\psi & \cos\theta\cos\psi + \\ -\sin\phi & \cos \end{bmatrix}$$

$$\begin{bmatrix} \sin^2\phi + \cos^2\phi\cos^2\psi + \cos^2\phi\sin^2\psi \\ -\cos\phi\sin\theta\sin\phi + (\cos\phi\sin\psi)(\cos\theta\cos\psi + \sin\theta\sin\phi\sin\psi) + (\cos\phi\cos\psi)(-\cos\theta\sin\psi + \sin\theta\cos\psi\sin\phi) \\ -\cos\theta\cos\phi\sin\phi + (\cos\phi\cos\psi)(\sin\theta\sin\psi + \cos\theta\cos\psi\sin\phi) + (\cos\phi\sin\psi)(-\sin\theta\cos\psi + \cos\theta\sin\phi\sin\psi) \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$