0.1 Determine the correct Euler transformation matrix for instruction sheet

(The symbols ϕ and θ are interchanged relative to the other file for Alan Brewer's notes)

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\phi & \sin\phi \\ 0 & -\sin\phi & \cos\phi \end{bmatrix} \quad \text{last, rotate about the new roll axis by } \phi$$

$$B = \begin{bmatrix} \cos\theta & 0 & -\sin\theta \\ 0 & 1 & 0 \\ \sin\theta & 0 & \cos\theta \end{bmatrix} \quad \text{second rotate about the new pitch axis by } \theta$$

$$C = \begin{bmatrix} \cos\psi & \sin\psi & 0 \\ -\sin\psi & \cos\psi & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad \text{first, rotate about the heading axis by } \psi$$

$$BC = \begin{bmatrix} \cos\theta\cos\psi & \cos\theta\sin\psi & -\sin\theta \\ -\sin\psi & \cos\psi & 0 \\ \sin\theta\cos\psi & \sin\theta\sin\psi & \cos\theta \end{bmatrix}$$

$$BC = D = \begin{bmatrix} \cos\theta\cos\psi & \cos\theta\sin\psi & -\sin\theta \\ -\sin\psi & \cos\psi & 0 \\ \sin\theta\cos\psi & \sin\theta\sin\psi & \cos\theta \end{bmatrix}$$

$$AD = \begin{bmatrix} \cos\theta\cos\psi & \cos\theta\sin\psi & -\sin\theta \\ -\cos\phi\sin\psi + \sin\theta\cos\psi\sin\phi & \cos\phi\cos\psi + \sin\theta\sin\phi\sin\psi & \cos\theta\sin\phi \\ \sin\phi\sin\psi + \cos\phi\sin\theta\cos\psi & -\cos\psi\sin\phi + \cos\phi\sin\theta\sin\psi & \cos\theta\cos\phi \end{bmatrix}$$

$$ABC = \begin{bmatrix} \cos\theta\cos\psi & \cos\theta\sin\psi & -\sin\theta \\ -\cos\phi\sin\psi + \sin\theta\cos\psi\sin\phi & \cos\phi\cos\psi + \sin\theta\sin\phi\sin\psi & \cos\theta\sin\phi \\ \sin\phi\sin\psi + \cos\phi\sin\theta\cos\psi & -\cos\psi\sin\phi + \cos\phi\sin\theta\sin\psi & \cos\theta\cos\phi \end{bmatrix}$$

$$\begin{bmatrix} \cos\theta\cos\psi & \cos\theta\sin\psi & -\sin\theta \\ -\cos\phi\sin\psi + \sin\theta\cos\psi\sin\phi & \cos\phi\cos\psi + \sin\theta\sin\phi\sin\psi & \cos\theta\sin\phi \\ \sin\phi\sin\psi + \cos\phi\sin\theta\cos\psi & -\cos\psi\sin\phi + \cos\phi\sin\theta\sin\psi & \cos\theta\cos\phi \end{bmatrix},$$

$$\text{transpose: } \begin{bmatrix} \cos\theta\cos\psi & -\cos\phi\sin\psi + \sin\theta\cos\psi\sin\phi & \sin\phi\sin\psi + \cos\phi\sin\theta\cos\psi \\ \cos\theta\sin\psi & \cos\phi\cos\psi + \sin\theta\sin\phi\sin\psi & -\cos\psi\sin\phi + \cos\phi\sin\theta\sin\psi \\ -\sin\theta & \cos\theta\sin\phi & \cos\theta\cos\phi \end{bmatrix}$$

Below, matrix times its transpose is verified to be the identity matrix

$$\begin{bmatrix} \cos\theta\cos\psi & \cos\theta\sin\psi & -\sin\theta \\ -\cos\phi\sin\psi + \sin\theta\cos\psi\sin\phi & \cos\phi\cos\psi + \sin\theta\sin\phi\sin\psi & \cos\theta\sin\phi \\ \sin\phi\sin\psi + \cos\phi\sin\theta\cos\psi & -\cos\psi\sin\phi + \cos\phi\sin\theta\sin\psi & \cos\theta\cos\phi \end{bmatrix} \begin{bmatrix} \cos\theta\cos\psi & -\cos\phi\sin \\ \cos\theta\sin\psi & \cos\phi\cos\psi \\ -\sin\theta & \mathrm{co} \end{bmatrix}$$

$$: \begin{bmatrix} \sin^2\theta + \cos^2\theta\cos^2\psi + \cos^2\theta\sin^2\psi \\ -\cos\theta\sin\theta\sin\phi + (\cos\theta\sin\psi)(\cos\phi\cos\psi + \sin\theta\sin\phi\sin\psi) + (\cos\theta\cos\psi)(-\cos\phi\sin\psi + \sin\theta\cos\psi\sin\phi \\ -\cos\theta\cos\phi\sin\theta + (\cos\theta\cos\psi)(\sin\phi\sin\psi + \cos\phi\sin\theta\cos\psi) + (\cos\theta\sin\psi)(-\cos\psi\sin\phi + \cos\phi\sin\theta\sin\psi \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$